Efficient Frequent Subgraph Mining in Transactional Databases

Pascal Welke

DSAA 2020

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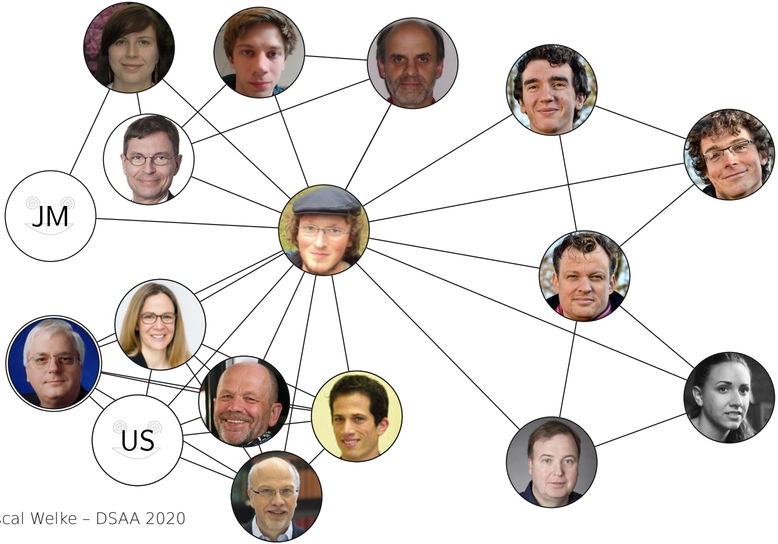
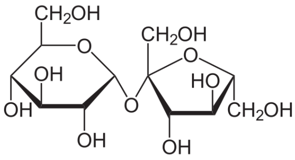
Example: Co-authorship Networks



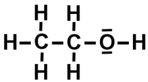
A *graph* $G = (V, E)$ consists of a set of vertices V and a set of edges E connecting pairs of vertices.

(Pictures taken from the home pages of the authors)

Example: Chemical Molecules



Saccharose

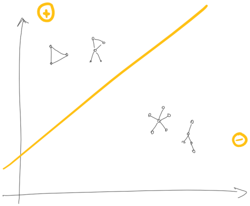


Ethanol

(commons.wikimedia.org)

How to Learn From Graphs?

- Similarity based learning methods
  - "close by objects behave similarly"



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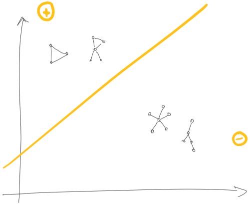

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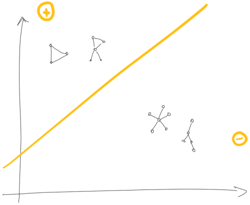
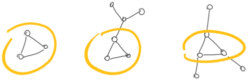


- Identification of relevant patterns



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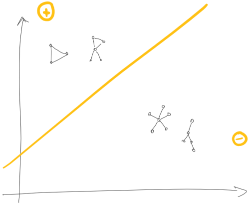
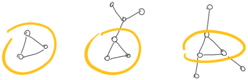


- Identification of relevant patterns
  - What is a pattern?
  - What is relevant?



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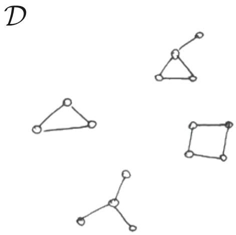
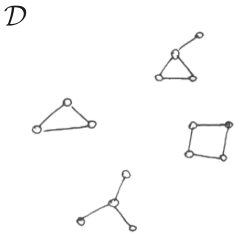


Frequent Subgraph Mining

- Let's say we have a few graphs (in a graph database $\mathcal{D}$)
- *Frequent subgraphs* are a reasonable choice to define similarities in a domain of graphs
  Deshpande et al (2005)



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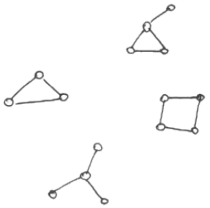

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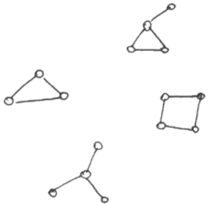


2-frequent subgraphs of $\mathcal{D}$

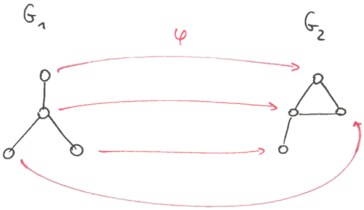
Definition

A *subgraph isomorphism* is an injective mapping

$$\varphi : V(G_1) \to V(G_2)$$

such that

$$(v_1, v_2) \in E(G_1) \Rightarrow (\varphi(v_1), \varphi(v_2)) \in E(G_2)$$



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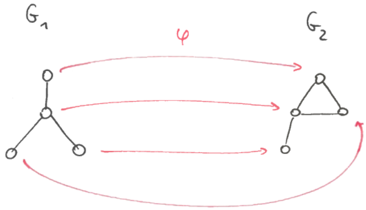

Deciding whether one exists, is *NP-hard*.

A brief history of exact FCSM algorithms

Frequent *Subtree* Mining

- FreeTreeMiner (Chi et al, 2003)
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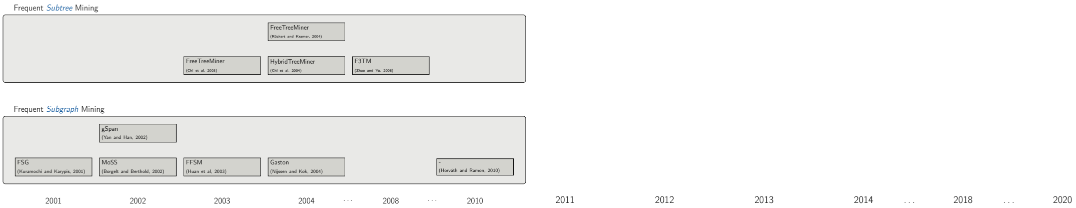

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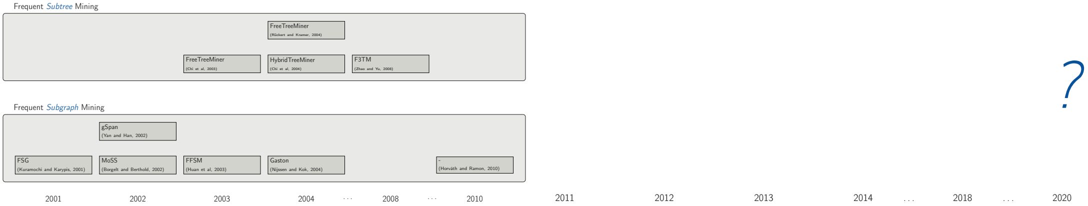
A brief history of exact FCSM algorithms

Efficient Frequent Subgraph Mining in Transactional Databases



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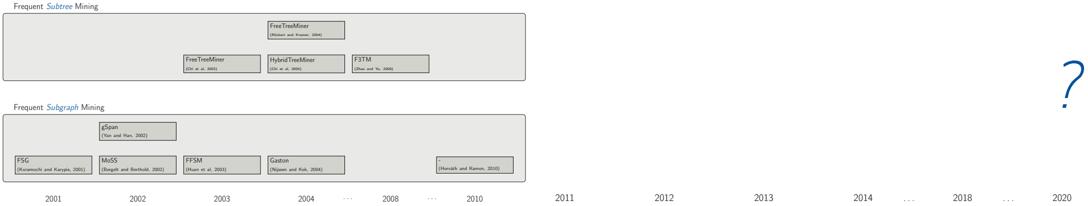

- All these methods enumerate the full set of frequent subtrees/subgraphs
- ...so why are we here, ten years later and discussing about this?

A quick quizz

- An acquaintance of yours has 200 graphs

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Nope! Tinnes (2020)

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   - a quick and general look at counting subgraph support
3. Some more current *inexact* solutions
   - *efficient* for *arbitrary* graph databases
   - though *incomplete*, comparable predictive performance to exact frequent subgraphs

Part 1.
Computational Complexity of Frequent Subgraph Mining

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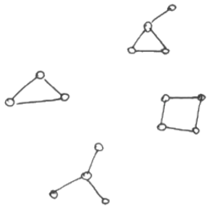

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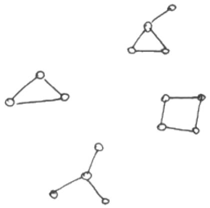

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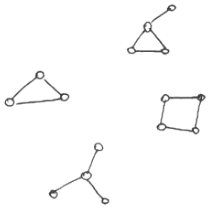

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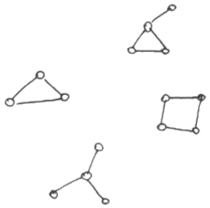

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(There is a bit more to it Horváth et al (2007))

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A few more detailed considerations

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- Incremental polynomial time
  - something in between

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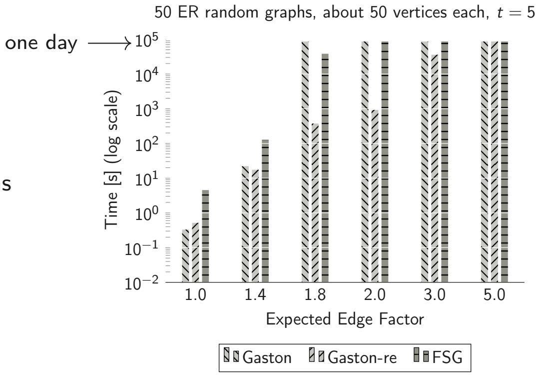

(Welke (2019))

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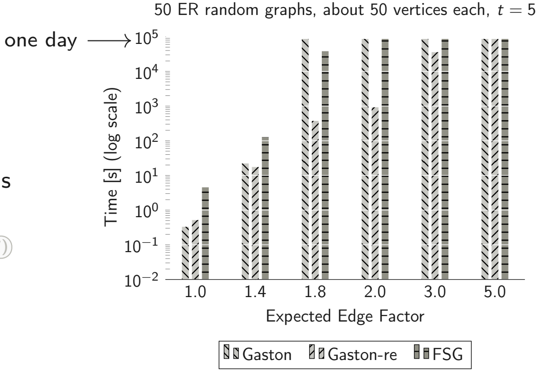

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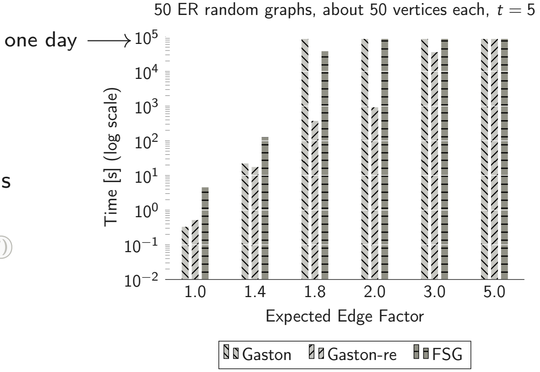

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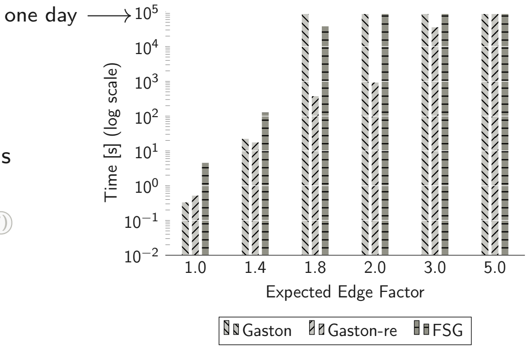
50 ER random graphs, about 50 vertices each, $t = 5$



(Welke (2019))

⇒ *There is no system that can reliably mine all frequent subgraphs for arbitrary graph databases of small to medium sized graphs*

Part 2.

Exact Frequent Subgraph Mining Algorithms (& Problems)

The Timeline, again...

Frequent *Subtree* Mining

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polynomial delay | incr. polynomial time | exponential in output size

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The Timeline, again...

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- not implemented: – (Horváth and Ramon, 2010)

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Part 3.

Efficient Inexact Mining Methods

1. Make the database simpler or smaller (Chen et al (2009)) (Welke et al (2019))

Possible Ways Out

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1. Make the database simpler or smaller (Chen et al (2009)) (Welke et al (2019))
2. Change the embedding operator (Li and Wang (2015)) (Schulz et al (2018))
3. Restrict the pattern language and allow one-sided error (Schulz et al (2018)) (Welke et al (2019)) (Welke et al (2020))

A More Recent Timeline

Frequent *Subtree* Mining

- –
  (Schulz et al, 2018)
- PS
  (Welke et al, 2018)
- BPS
  (Welke et al, 2019)
- HOPS
  (Welke et al, 2020)

Frequent *Subgraph* Mining

- SUMMARIZE-MINE
  (Chen et al, 2009)
- MUSE
  (Zou et al, 2010)
- REAFUM
  (Li and Wang, 2015)

2009 2010 ··· 2015 ··· 2018 2019 2020

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Conclusion & Outlook

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- Can we combine exact and approximate tools to achieve "one-size-fits-all" mining?
- Can we devise efficient methods for non-tree patterns?

References I

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